

# Excitation of self-induced surface states in parabolic geometry



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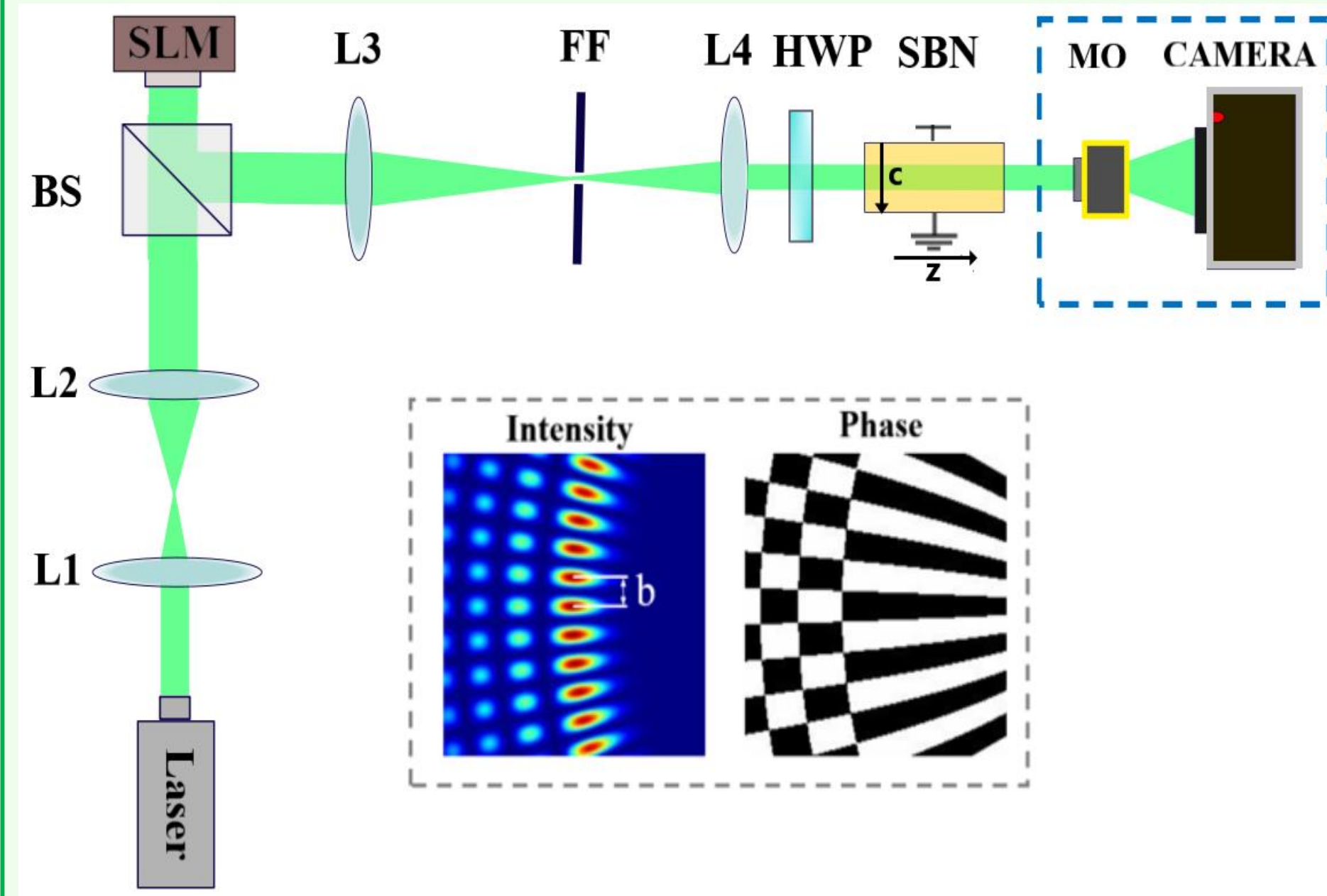
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Nonlinear  
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**Abstract:** We report the observation of parabolic surface states during the nonlinear propagation of Weber beams in an SBN crystal. The refractive index is modulated anisotropically using the optical induction technique, without the need for a pre-inscribed lattice. These self-induced surface states can be controlled by tuning the Weber beam's scale, parabolicity, orientation, and power. Additionally, we observe oscillatory discrete surface states under linear propagation of a Gaussian probe beam in an aperiodic Weber photonic lattice. In both regimes, surface states manifest either as extended states across multiple adjacent parabolas or as localized edge states along the outermost parabola. This represents the first demonstration of self-induced parabolic surface states in aperiodic systems.

## Experimental setup for studying nonlinear propagation of Weber beams



**Fig. 1. Experimental setup**

L: lens  
BS: beam splitter  
SLM: spatial light modulator  
FF: Fourier filter  
SBN: crystal  
HWP: half-wave plate  
MO: microscope objective

Inset: Example of Weber beam intensity and phase profiles. Characteristic beam width is labeled  $b$ .

## Modeling of Weber beams propagation in photorefractive medium

Anisotropic approximation is used to compute the refractive index modulation by solving nonlinear Schrödinger equation and potential equation:

$$i\partial_z A(r) + \frac{1}{2k_z} [\Delta_{\perp} + G(I)] A(r) = 0$$

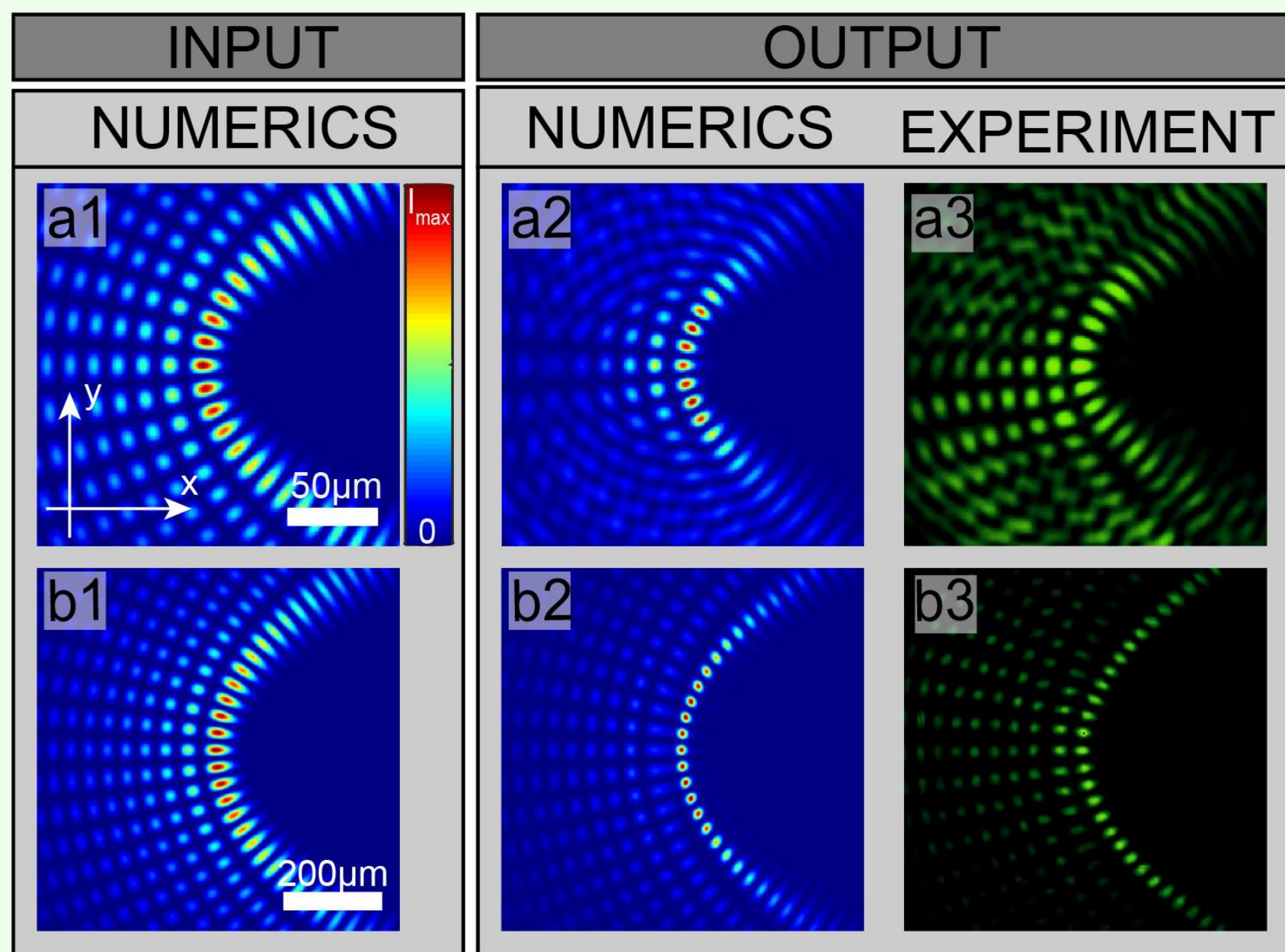
$$\Delta_{\perp} \Phi_{sc} + \nabla_{\perp} (1 + I) \cdot \nabla_{\perp} \Phi_{sc} = E_{ext} \partial_x \ln(1 + I)$$

The paraxial scalar light field  $A$  is represented by even Weber beams defined as:

$$A = U_e(\eta, \xi; a) = \frac{1}{\pi\sqrt{2}} |\Gamma_1|^2 P_e(\sigma\xi; a) P_e(\sigma\eta; -a)$$

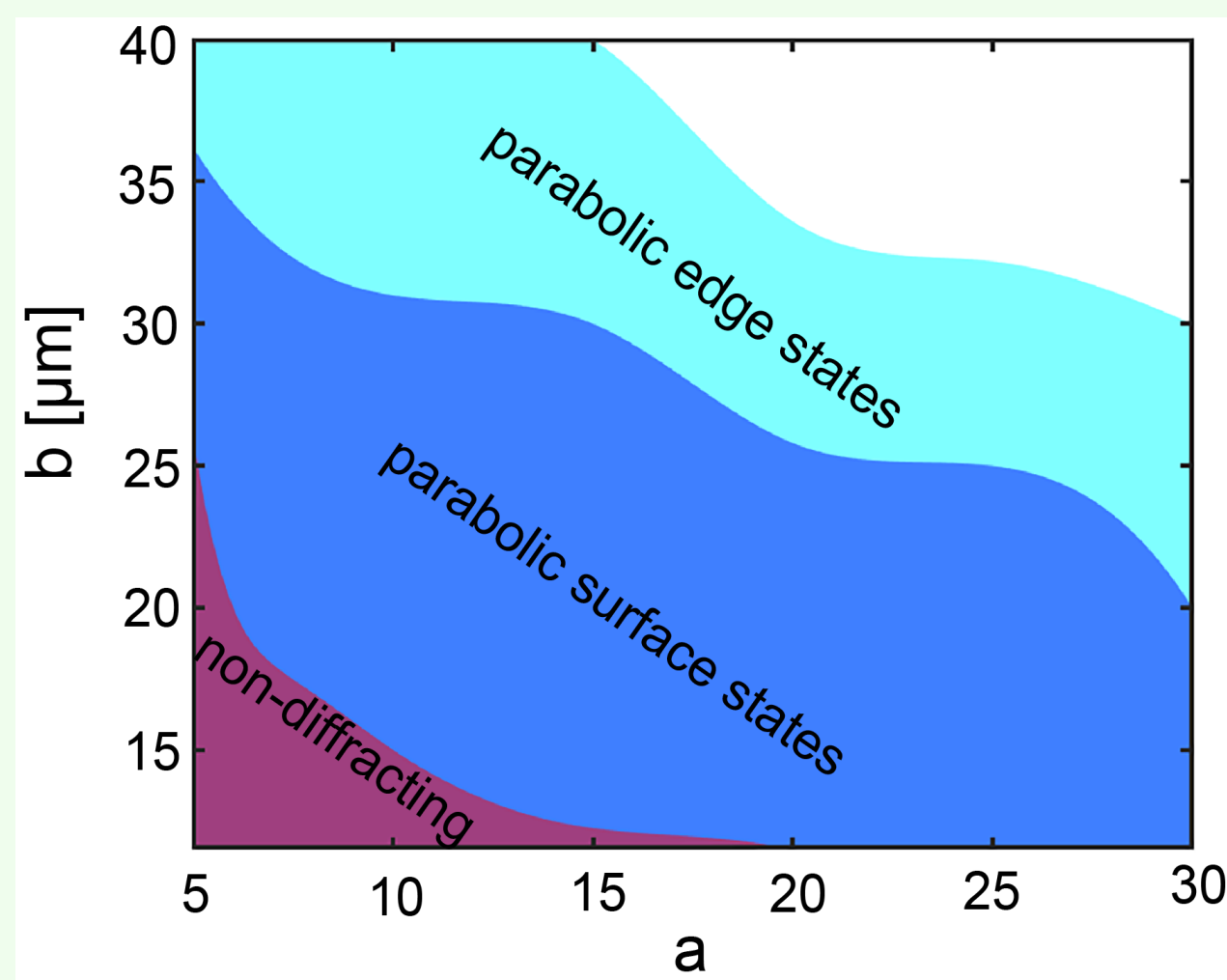
$P_e$  denotes the even parabolic cylinder function,  $\Gamma$  is the Gamma function,  $\Gamma_1 = \Gamma_1[\frac{1}{4} + \frac{1}{2}ia]$ ,  $\sigma = (4\pi/\lambda)^{1/2}$ .

## Self-induces parabolic surface states formed during nonlinear Weber beam propagation



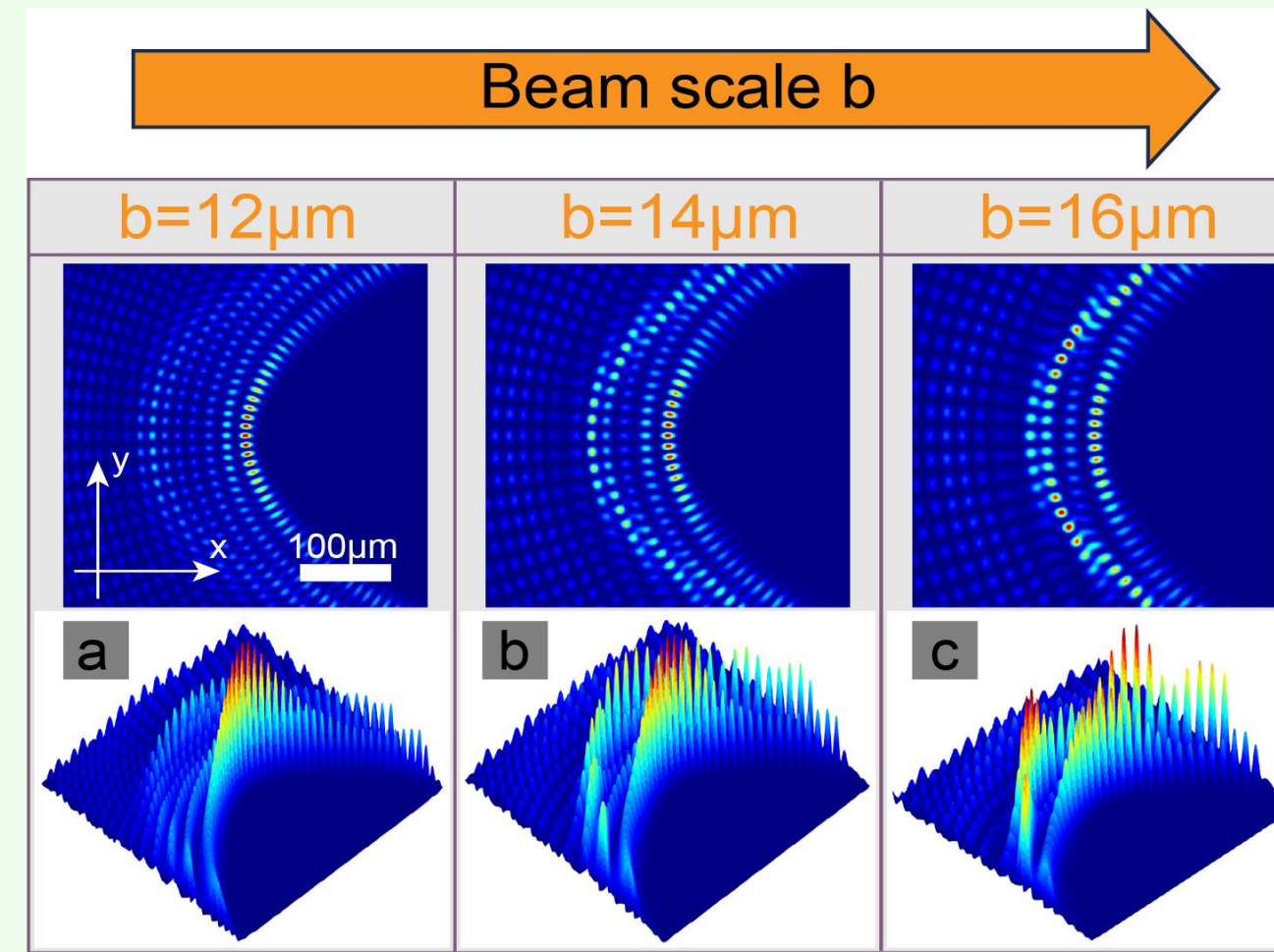
**Fig. 2.** Examples of self-induced parabolic surface states. First column: Input Weber beam intensity. Second column: Numerical output at the exit face the crystal. Third column: Experimental output after 20 mm at the exit face the crystal. Parameters:

(a)  $a = 6$ ,  $b = 12 \mu\text{m}$ ,  $I = 0.7$ ,  $P_{\omega} = (34 \pm 1.02) \mu\text{W}$   
(b)  $a = 10$ ,  $b = 35 \mu\text{m}$ ,  $I = 0.08$ ,  $P_{\omega} = (3.4 \pm 0.102) \mu\text{W}$ .

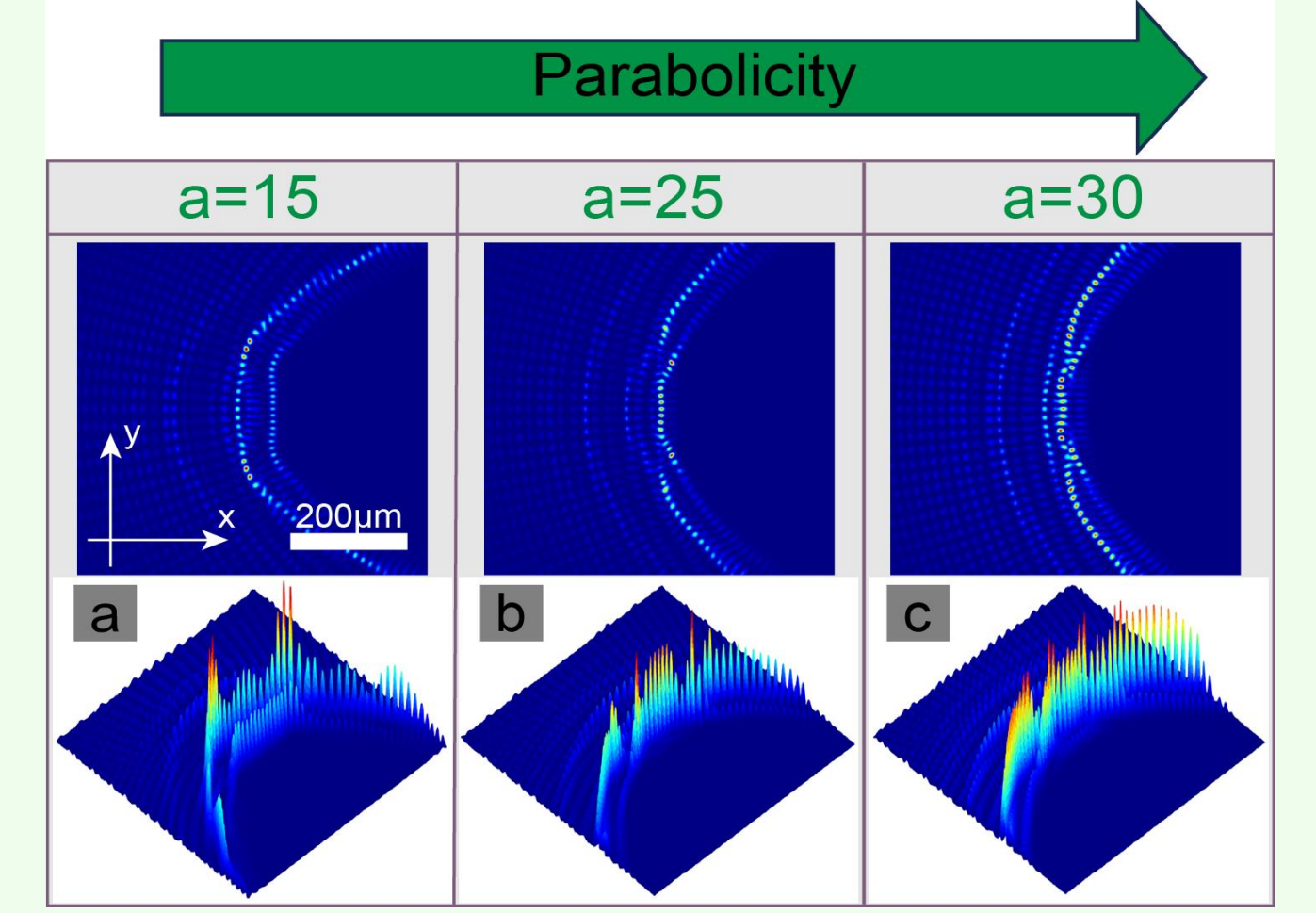


**Fig. 3.** Parameter space diagram showing distinct propagation regimes of Weber beams in the nonlinear regime.  $b$  represents the beam scale, and  $a$  defines the beam's parabolicity. Input intensity:  $I = 0.16[a.u.]$ .

## Influence of Weber beam characteristics on the surface states formation

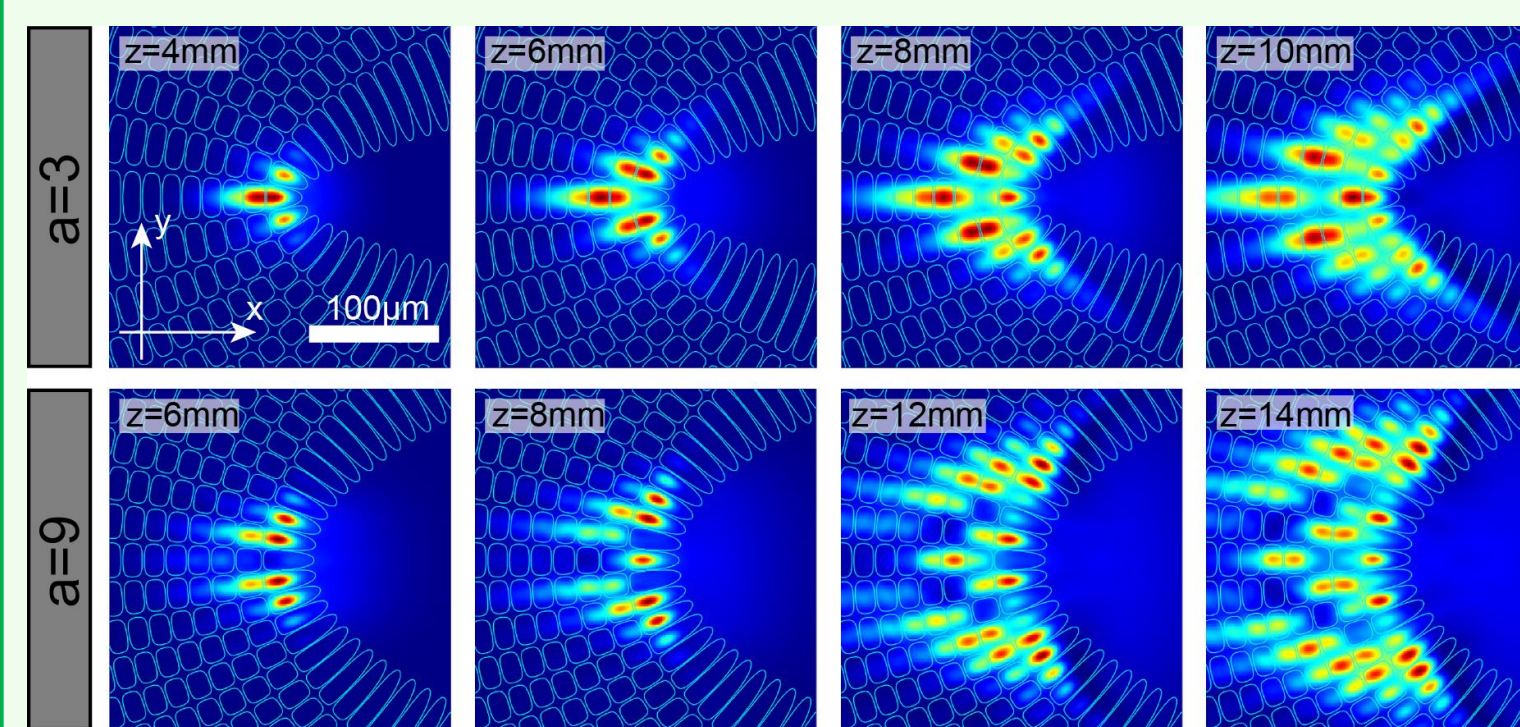


**Fig. 4.** Effect of Weber beam scale  $b$  on the formation of parabolic surface states. Shown are transverse intensity profiles after 20 mm of propagation. Parameters: Weber beam parabolicity  $a = 15$ , and input intensity  $I = 0.3$ .

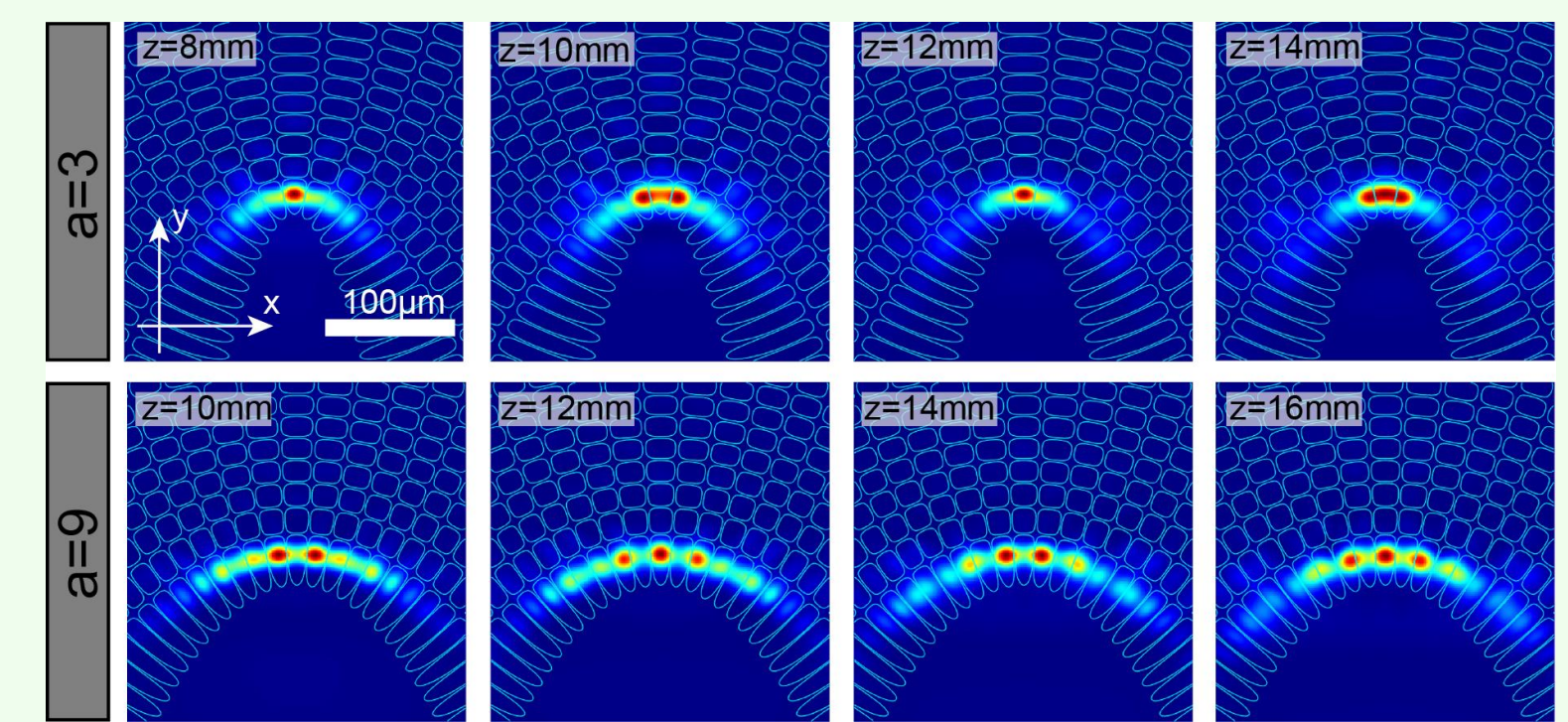


**Fig. 5.** Effect of Weber beam parabolicity  $a$  on the formation of parabolic surface states. Shown are transverse intensity profiles after 20 mm propagation. Parameters: beam scale  $b = 15 \mu\text{m}$ , input intensity  $I = 0.8$ .

## Oscillatory surface states obtained during linear propagation of Gaussian probe beam in Weber lattice



**Fig. 6.** Discrete surface states in an aperiodic Weber lattice. Transverse intensity profiles of a Gaussian probe beam at different propagation distances. Rows correspond to different lattice parabolicities:  $a = 3$  (first row),  $a = 9$  (second row). Contours indicate lattice structure. Parameters:  $b = 15 \mu\text{m}$ ,  $I_{latt} = 0.5$ .



**Fig. 7.** Discrete edge states in an aperiodic Weber lattice. Shown are transverse intensity profiles of a Gaussian probe beam at different propagation distances. Rows correspond to lattice parabolicities:  $a = 3$  (first row),  $a = 9$  (second row). Contours outline the Weber lattice structure. Parameters:  $b = 12 \mu\text{m}$ ,  $I_{latt} = 0.5$ .

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